Online Appendix for "Why is productivity slowing down?"

Ian Goldin[1], Pantelis Koutroumpis[1], François Lafond[2], and Julian Winkler[1]

[1]*University of Oxford*

Appendix

A Additional results on the aggregate slowdown

A.1 Slowdown and missing GDP

In the spirit of Syverson (2017), to highlight the consequences of slowing productivity, we compute what GDP would have been, should the productivity slowdown not have occurred. Formally, labor productivity growth between two periods t and $t-1$ is

$$g_t \equiv \Delta \log y_t = \log \frac{Y_t}{L_t} - \log \frac{Y_{t-1}}{L_{t-1}},$$

where Y denotes real value added (labeled `VA_Q` by KLEMS), L units of labor, in our case hours worked (`H_EMP`), and Δ is the first difference operator. KLEMS 2019 provides the labor productivity growth rates (`LP1_G`) directly for our five countries, although certain years may be missing in specific instances. Since the data for (Y/L) generally starts in 1995, the first observed growth rates in France, Germany and the United Kingdom are in 1996. However, data on productivity growth rates for the United States only start from 1998, while in Japan there is an observation for 1995. The data for labor productivity growth extend to 2017, with another exception for Japan, for which data end in 2015.

With these caveats in mind, we compute labor productivity growth for our five countries by averaging the growth rates across all years in the base period (0), 1996-2005, using

$$g_{(0)} \equiv \frac{1}{T_{(0)}} \sum_{t=1996}^{2005} g_t, \tag{9}$$

where $T_{(0)} = 2005 - 1996 + 1 = 10$. These values are listed in the first column of Table 1, multiplied by 100 to denote percentage points (pp).

For the period 2006-2017 (or 2015 in the case of Japan), denoted (1), the *realized* average rate of labor productivity growth is defined as

$$g_{(1)} \equiv \frac{1}{T_{(1)}} \sum_{t=2006}^{2017} g_t, \tag{10}$$

where $T_{(1)} = 2017 - 2006 + 1 = 12$. In the third column, we compute the slowdown in labor productivity growth as the difference between the two average growth rates,

$$\text{Slowdown} \equiv g_{(0)} - g_{(1)}.$$

In column four, we calculate GDP per capita in 2017 (2015 for Japan) as

$$\text{GDP per capita}_{2017} \equiv \frac{Y_{2017}}{N_{2017}}, \tag{11}$$

where N_{2017} is the mid-year population count taken from the Conference Board's Total Economy Database^{TM} (The Conference Board 2020) for each country[24].

[24] https://conference-board.org/data/economydatabase, version July 2020.

To calculate "missing GDP", in column five, we write a counterfactual GDP per capita in 2017 in terms of a counterfactual level of labor productivity,

$$\frac{\tilde{Y}_{2017}}{N_{2017}} = \frac{Y_{2017}}{N_{2017}} \frac{\tilde{Y}_{2017}}{Y_{2017}} = \frac{Y_{2017}}{N_{2017}} \frac{\tilde{y}_{2017}}{y_{2017}}, \tag{12}$$

using a tilde to denote a variable's "counterfactual" and because $\tilde{L} = L$. The labor productivity levels were the same until 2005, so

$$\frac{\tilde{y}_{2017}}{y_{2017}} = \frac{y_{2005} \ \exp\left(g_{(0)} T_{(1)}\right)}{y_{2005} \ \exp\left(g_{(1)} T_{(1)}\right)} = \exp\left((g_{(0)} - g_{(1)}) T_{(1)}\right),$$

since $\tilde{g}_{(1)} = g_{(0)}$ is the growth rate that would have happened had labor productivity growth continued on its 1996-2005 trend after 2005. Substituting this back into Eq. 12, we have

$$\frac{\tilde{Y}_{2017}}{N_{2017}} = \frac{Y_{2017}}{N_{2017}} \exp\left((g_{(0)} - g_{(1)}) T_{(1)}\right), \tag{13}$$

Taking stock, we compute the "missing GDP" in 2017 (again, 2015 for Japan) as

$$\text{Missing GDP per capita}_{2017} \equiv \frac{\tilde{Y}_{2017}}{N_{2017}} - \frac{Y_{2017}}{N_{2017}} = \frac{Y_{2017}}{N_{2017}} \left[\exp\left((g_{(0)} - g_{(1)}) T_{(1)}\right) - 1\right],$$

using Eq. 13, and the values defined in Eqs. 9,10 and 11. Finally, note that volume indices for real GDP are indexed to 2010 prices in KLEMS. We convert GDP and "missing GDP" per capita to 2017 prices by multiplying by the price index (VA_PI) for 2017, divided by 100, or equivalently by taking directly nominal values (VA), instead of volumes in 2010 prices. The results are in the fifth column of Table 1.

In principle, for the purpose of constructing Table 1, using data from the Total Economy Database^{TM} would have been better due to its larger time coverage. We have constructed such a table and it does not change our narrative very much. The slowdown (2006-2017 compared to 1996-2005, for all countries) ranges from 0.68 (Germany) to 1.77 (UK). We prefer to use EU KLEMS 2019 throughout the paper for consistency.

A.2 Long term trends and convergence

Productivity slowed down in advanced economies when comparing post 2005 against 1996-2005. For Europe, can this simply reflect the fact that Europe's productivity growth has declined for decades as it achieved convergence to the US? And for the US, can this simply reflect the fact that 1996-2005 was an exceptional decade, and the US is now back to a more "normal" rate of productivity growth?

Let us start with convergence. Figure 6 shows that European countries and Japan had converged or stopped converging after 1990. The growth rates of all economies were more or less synchronized after this. Thus, the slowdown in Europe between 1996-2005 and 2006-2017 is not due to a lower contribution of convergence factors.

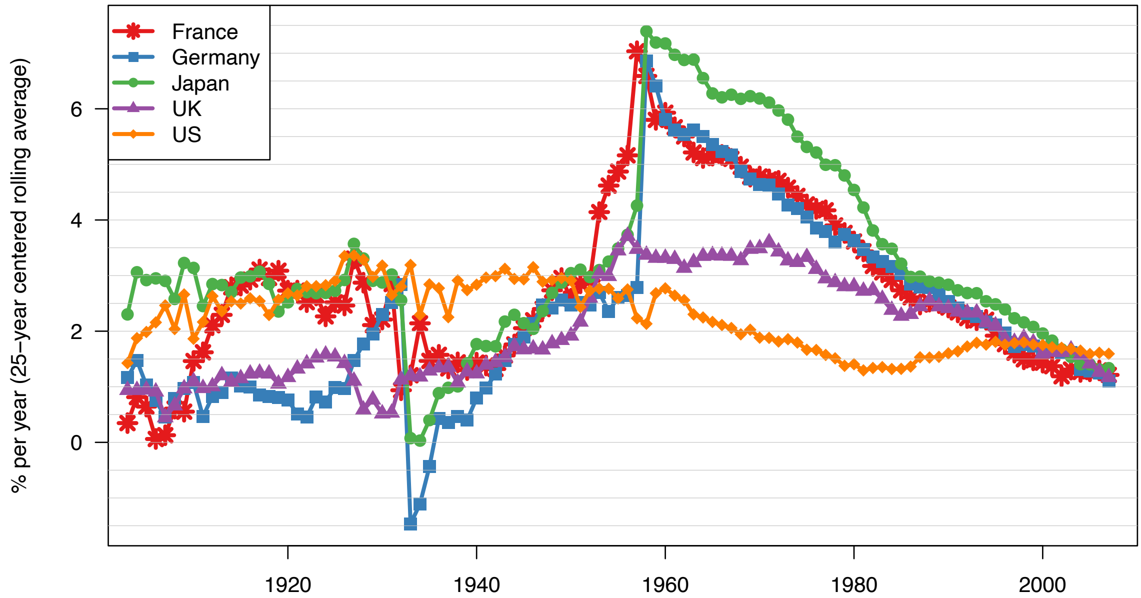

Figure 5: Long term trends in labor productivity. Data from the Long-Term Productivity Database (Bergeaud et al. 2021).

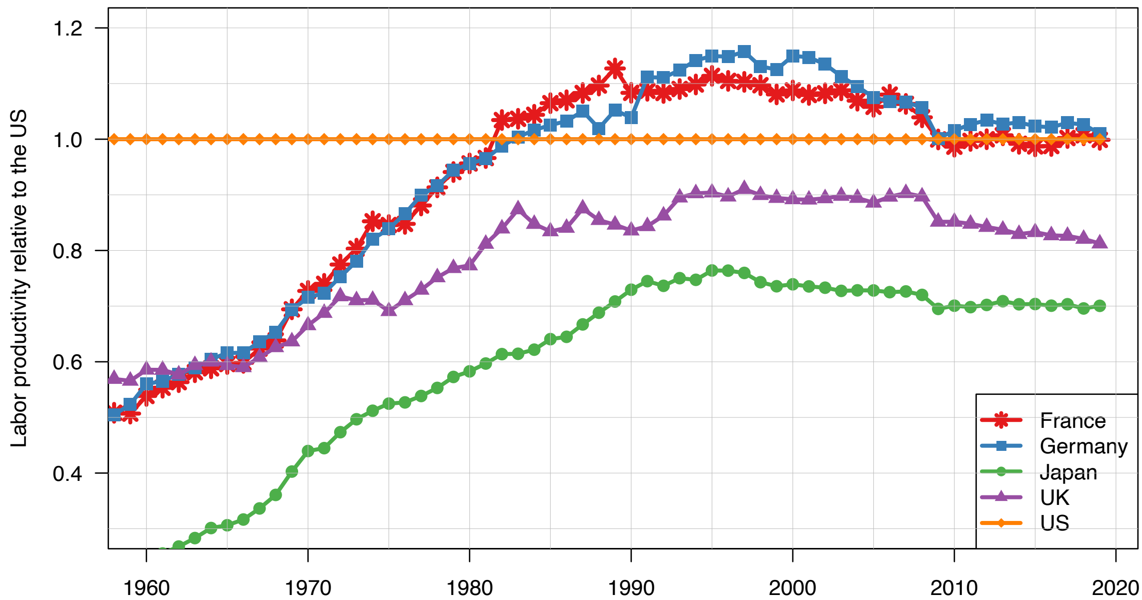

Figure 6: Convergence of productivity levels. Data from the Long-Term Productivity Database (Bergeaud et al. 2021).

How about long term trends? Although recent work has been able to highlight differences across countries and periods (Fouquet & Broadberry 2015), very long run historical data suggest very small growth rates on average, with the industrial revolution being exceptional. It is possible that growth is a succession of adjustments in levels and that most of the low-hanging fruit has been reaped, so lower growth simply reflects the end of these adjustments in levels. But there are also good reasons to think that endogenous growth is possible, through well-documented mechanisms of non-rival knowledge accumulation (Romer 1986). Throughout the paper, we attempt to discuss whether a specific mechanism for the productivity slowdown corresponds to a weakening of "long-run", "permanent" growth rates, or to level effects running off – but it is very complex and we do not claim to have resolved this issue.

Having said that, we can look at labor productivity growth rates over the past century, thanks to the data from Bergeaud et al. (2016, 2021), see Figure 5. Table 13 shows average labor productivity growth rates for subperiods. It is clear that the last period features particularly low productivity growth rates, even for the US. Rates of productivity growth in the range [0.5-1]% have been rare, especially in such a pervasive fashion as in the last decade.

	1891-1910	1911-1930	1931-1950	1951-1970	1971-1990	1991-2005	2006-2018
France	1.21	3.39	0.78	5.36	3.33	1.89	0.68
Germany	1.75	0.73	0.02	5.82	3.21	2.27	0.70
Japan	2.16	2.69	1.07	7.32	3.87	2.03	0.71
UK	0.74	1.46	1.16	3.46	2.48	2.43	0.47
US	1.43	2.78	3.22	2.48	1.34	2.05	1.06

Table 13: Average growth rates of labor productivity ($US 2010 PPP per hour worked), for several long periods. Data from the Long-Term Productivity Database (Bergeaud et al. 2016).

To sum up, we contend that that there is still, indeed, a productivity slowdown. There is some merit in the argument that all five advanced economies are now more or less at the frontier, and that low growth rates at the frontier are "normal". But the particularly low rates observed (less than 1%), in a context of very salient technological transformations, merits a detailed investigation, as we attempt to provide here.

Finally, one can ask: is productivity still slowing down? Is there evidence of a negative trend *within* the "slow decade"? Figure 1 and standard regressions do not suggest that this is the case, but we refrain from investigating this further.

B Conceptual framework

The paper computes estimates of the mismeasurement bias, as well as other explanations of the productivity slowdown. In the absence of a complete theoretical framework that would lead to a precise and additive decomposition, we are unable to identify the extent to which the various effects that we report "overlap".

In this Appendix, we make some progress for one specific estimate: mismeasurement. If GDP growth is mismeasured, does it imply that all terms of the growth accounting decomposition are mismeasured, and if so, in which proportion?

For this Appendix let us adopt the shorthand $\hat{x} \equiv \Delta \log x$, also omitting the subscript t. Let TFP be measured from observed data as in Eq. 1

$$\hat{a} = \hat{y} - \alpha\hat{h} - (1-\alpha)\hat{k}, \tag{14}$$

where α is the labor share

$$\alpha \equiv \frac{wL}{PY}. \tag{15}$$

We assume that true output grows faster than measured output

$$\hat{y}^* = \hat{y} + \mathcal{B}, \qquad \mathcal{B} > 0, \tag{16}$$

but we do not know a priori the sign of the bias for capital deepening (see also Crouzet & Eberly (2021)),

$$\hat{k}^* = \hat{k} + \mathcal{D}, \qquad D \lesseqqgtr 0. \tag{17}$$

The assumptions above are motivated by what we discuss in the mismeasurement Section (3): part (and only part) of the output mismeasurement is due to mismeasurement of investment, either because intangible investment is wrongly treated as intermediate, or because the deflators for investment goods are biased.

Now, can we express measured TFP as a function of true TFP and a mismeasurement bias? True TFP is defined as

$$\hat{a}^* = \hat{y}^* - \alpha^* \hat{h} - (1 - \alpha^*)\hat{k}^*, \tag{18}$$

where the labor share is defined using true output,

$$\alpha^* \equiv \frac{wL}{P^* Y^*}. \tag{19}$$

This assumes that labor income wL is always well measured[25] even when output is mismeasured, as assumed in Crouzet & Eberly (2021). Now, inserting the definition of true output (16) and true capital deepening (17) in the definition of true TFP (18), solving it for $\hat{y}$ and substituting the definition of observed TFP (14), we find

$$\hat{a} = \hat{a}^* - \mathcal{B} + \mathcal{C}, \tag{20}$$

where

$$\mathcal{C} \equiv (\alpha - \alpha^*)(\hat{k} - \hat{h}) + (1 - \alpha^*)\mathcal{D}. \tag{21}$$

Eqs. 20-21 show that we cannot simply remove the mismeasurement bias $\mathcal{B}$ from observed TFP $\hat{a}$, because of the term $\mathcal{C}$, which includes two independent terms: first because of mismeasurement of capital deepening, and second because mismeasurement of output implies mismeasurement of the labor share (even if there was no mismeasurement of capital deepening ($\mathcal{D} = 0$)).

So, can we ignore $\mathcal{C}$, and report all the mismeasurement bias as an explanation for the TFP slowdown only? First of all, the term $(1 - \alpha^*)\mathcal{D}$ may not be significant, because even if investment is mismeasured, the effect on the growth rate of capital deepening is ambiguous, so that $\mathcal{D}$ may not be large, let alone change very much between the two decades. For instance, a potential source of mismeasurement of the growth rate of capital services would be a mismeasurement of intangibles. This would affect $\alpha - \alpha^*$ as well as $\mathcal{D}$. Fortunately, EU-KLEMS 2019, Corrado et al. (2016) and Crouzet & Eberly (2021) have looked into this in detail. There is indeed a bias, but it is small. If we consider the growth accounting results performed using EU-KLEMS's intangibles-extended accounts, Table 17, we find that the contribution of TFP to the slowdown is very similar to what it is using national accounts data, Table 2. In the main text (Table 5) we report the bias due to the mismeasurement of intangibles additively, with no discussion of whether it overlaps with the biases to the deflators; this simply reflects our view that uncertainty around the biases themselves is far larger than these overlaps.

[25] which is not true if some ambiguous income is wrongly attributed to capital, as suggested by Koh et al. (2020).

Second, the factor $(\hat{k}-\hat{h})$ in the first term may indeed be important to us, because $\hat{h}$ is usually small, so the slowdown in $\hat{k}$ will translate almost one-to-one into a slowdown of $(\hat{k}-\hat{h})$. The key question then is whether $(\alpha-\alpha^*)$ is large. A crucial point is that the difference between α and α^* comes from the mismeasurement of *nominal* income. As we have seen in Section 3, there are indeed uncertainties with the GDP boundary, so it is conceivable that nominal income is mismeasured. But the key issue here is the mismeasurement of the labor share, so the main relevant source of mismeasurement is mismeasurement of output that would not simultaneously affect measurement of labor income. For instance, according to the SNA guidelines, statistical agencies are supposed to evaluate the informal economy by running household surveys to understand not only how much output is missing, but also the hours worked, the number of employees and their skills. If they miss part of output, they would also miss part of labor income, so the effect on the mismeasurement of the labor share is ambiguous and unlikely to be high.

Eventually, and for simplicity, Tables 11 and 12 report mismeasurement as an explanation for the TFP slowdown only, although the discussion above suggests that part of mismeasurement should change the contribution of capital deepening.

The last step to get Eq. 3 is that we further assume that true TFP growth $\hat{a}^*$ can be split into allocative efficiency and "Technology" – see Section 7.4. This leads to (from Eq. 20)

$$\hat{a} = \hat{a}^{\text{alloc}} + \hat{a}^{\text{tech}} - \mathcal{B}. \tag{22}$$

Substituting Eq. 22 into the standard growth accounting equation (Eq. 1), and switching back to the notation $\Delta \log x = \hat{x}$ gives Eq. 3 in the main text.

C Additional results on labor productivity decompositions

C.1 Evidence from other studies

Table 14 synthesizes the results of existing growth accounting studies on the recent productivity slowdown. Not all studies use comparable breakdowns in years; many, for example, will compare productivity growth pre- and post-2007, instead of 2005. Not all studies use comparable data on inputs either: notable differences emerge when calculating the contributions of labor composition or ICT capital in isolation. We make an arbitrary judgement on the contribution to the slowdown based on the result of a given paper and a given input, from high (++), modest (+), negligible (0), to worsening (–) the slowdown. When a given input does not feature in the study, we leave the entry blank; this means that a study which only considers non-ICT capital growth will have the corresponding entry filled, even though their aggregate capital measure may well include ICT capital, which we can only leave blank. We also record the data used by these various studies, as well as other idiosyncrasies, such as country aggregates that often appear for European countries.

Broadly speaking, Table 14 confirms our results in Section 2: TFP is the main source of the slowdown, except in Japan, while capital deepening is also important, but labor composition is not found to explain much.

	Data	Labor Composition	Non-ICT Capital	ICT Capital	TFP
France					
van Ark (2016*a*)	TED	+	+	+	++
Cette et al. (2016)	LTPD			0	++
Fernald & Inklaar (2020)	KLEMS 17,12[b], PWT[c]	0/–	+/0[d]		++
Gordon & Sayed (2019)	KLEMS 17,12[a]	–	++	0	++
Oulton (2019)	KLEMS 17		++		++
Inklaar et al. (2019)	KLEMS 17		+		++
Germany					
van Ark (2016*a*)	TED	–	+	–	++
Cette et al. (2016)	LTPD			0	++
Baily et al. (2020)	OECD		++		+
Fernald & Inklaar (2020)	KLEMS 17,12[b], PWT[c]	0/–	+/0[d]		++
Gordon & Sayed (2019)	KLEMS 17,12[a]	–	++	0	++
Oulton (2019)	KLEMS 17		++		++
Inklaar et al. (2019)	KLEMS 17		+		0
Japan					
Baily et al. (2020)	OECD		++		+
Jorgenson et al. (2018)	KLEMS, NSA		++		–
UK					
Riley et al. (2018)	NSA	–	+		++
Goodridge et al. (2018)	NSA	–	+[e]		++
Tenreyro (2018)	NSA	0		++	++
van Ark (2016*a*)	TED	+	+	+	++
Cette et al. (2016)	LTPD			+	++
Fernald & Inklaar (2020)	KLEMS 17,12[b], PWT[c]	0/–	+/0[d]		++
Gordon & Sayed (2019)	KLEMS 17,12[a]	–	++	0	++
Oulton (2019)	KLEMS 17		++		++
Inklaar et al. (2019)	KLEMS 17		++		++
US					
Baily & Montalbano (2016)	NSA	0	++		++
Murray (2018)	NSA	0	+		++
van Ark (2016*a*)	TED	+	+	+	++
Cette et al. (2016)	LTPD			+	++
Baily et al. (2020)	OECD		+		++
Gordon & Sayed (2019)	KLEMS 17,12	0	++	+	++
Oulton (2019)	KLEMS 17		++		++
Inklaar et al. (2019)	KLEMS 17		++		++

[a] Aggregated as EU-10 [b] Aggregated as EU-8 [c] Aggregated as EU-15 [d] Calculated as the capital-output ratio [e] A separate intangible capital term yielded a negligible (0) contribution.
Data sources, and their shorthands, are: one or more country-specific national statistical agencies (NSA), Total Economy Database (TED), Long Term Productivity Database (LTPD), Penn World Tables (PWT), OECD Statistics (OECD), and various vintages of EU KLEMS (KLEMS 1X). The contributions of proposed sources to the slowdown are denoted by a symbol; high (++), modest (+), negligible (0), worsening (–). A missing component within a paper is reflected by a blank entry.

Table 14: Proposed sources for the labor productivity growth slowdown from 13 growth accounting studies with diverse data sources

C.2 Contributions of TFP and capital deepening using the OECD's Productivity database

		$\Delta \log y_t$	$\Delta \log A_t$	$(1-\alpha_t)\Delta \log k_t$
France	1995-2005	1.74	0.96	0.77
	2006-2017	0.71	0.15	0.55
	Slowdown	1.03	0.80	0.22
	Share	1.00	0.78	0.21
Germany	1995-2005	1.54	0.79	0.74
	2006-2017	0.87	0.63	0.23
	Slowdown	0.68	0.17	0.51
	Share	1.00	0.24	0.76
Japan	1995-2005	2.11	0.84	1.25
	2006-2017	0.75	0.48	0.27
	Slowdown	1.36	0.36	0.98
	Share	1.00	0.27	0.72
United Kingdom	1995-2005	2.22	1.73	0.45
	2006-2017	0.47	0.09	0.37
	Slowdown	1.75	1.64	0.08
	Share	1.00	0.94	0.05
United States	1995-2005	2.27	1.36	0.88
	2006-2017	1.06	0.46	0.59
	Slowdown	1.21	0.90	0.29
	Share	1.00	0.75	0.24

Table 15: Growth accounting results using OECD Productivity data.

Table 15 shows the results. The most noticeable difference with EU KLEMS is the substantially smaller slowdown of capital deepening for the US. The OECD data (OECD 2021*c*) slightly mitigates the result from KLEMS that the source of the slowdown is only TFP in France and only capital deepening in Japan.

C.3 Contributions of industries and reallocation using the OECD's STAN

	Total	Manufacturing	Wholesale, Retail and Repair	Financial and Insurance Activities	Information and Communication	Other	Reallocation
France							
1996-2005	1.61	0.62	0.16	0.09	0.23	0.49	0.02
2006-2015	0.67	0.27	0.09	0.06	0.11	0.21	-0.08
Slowdown	0.94	0.35	0.07	0.03	0.11	0.28	0.10
Share	1.00	0.37	0.07	0.03	0.12	0.30	0.11
Germany							
1996-2005	1.87	0.69	0.31	-0.08	0.17	0.45	0.33
2006-2015	0.87	0.39	0.16	0.06	0.21	0.14	-0.09
Slowdown	1.01	0.30	0.15	-0.13	-0.04	0.31	0.42
Share	1.00	0.30	0.15	-0.13	-0.04	0.30	0.42
United Kingdom							
1996-2005	2.18	0.50	0.17	0.27	0.31	0.66	0.27
2006-2015	0.48	0.14	0.18	0.00	0.08	-0.18	0.26
Slowdown	1.70	0.36	-0.01	0.27	0.23	0.84	0.01
Share	1.00	0.21	-0.01	0.16	0.14	0.50	0.01
United States							
1996-2005	2.36	0.91	0.56	0.32	0.27	0.40	-0.10
2006-2015	0.97	0.25	0.08	0.09	0.22	0.45	-0.11
Slowdown	1.39	0.66	0.49	0.24	0.05	-0.05	0.01
Share	1.00	0.47	0.35	0.17	0.03	-0.03	0.01

Table 16: Industry decomposition for the slowdown in labor productivity growth pre- and post-2005 using OECD's STAN data.

As a robustness check for the industry level decomposition in labor productivity growth, we reproduce the decomposition using data from the OECD's STAN database (OECD 2021*a*), in Table 16. The downside is that hours worked data for Japan are missing, and the productivity series generally do not extend beyond 2015. Despite these shortcomings, results from the industry-level decomposition, using the same method of Tang & Wang (2004), are almost identical to those derived from the KLEMS 2019 data, visible in Table 3.

C.4 Contribution of factors and TFP using KLEMS intangibles-augmented database

		$\Delta \log y_t$	$\Delta \log A_t$	$(1-\alpha_t)\Delta \log k_t$	$\alpha_t \Delta \log h_t$
France	1996-2005	1.70	1.20	0.20	0.30
	2006-2017	0.75	0.20	0.15	0.40
	Slowdown	0.95	1.00	0.05	-0.09
	Share	1.00	1.05	0.05	-0.10
Germany	1996-2005	1.88	1.12	0.61	0.15
	2006-2017	0.92	0.86	0.08	-0.03
	Slowdown	0.96	0.26	0.53	0.17
	Share	1.00	0.27	0.55	0.18
Japan	1996-2005	1.75	0.14	1.29	0.33
	2006-2015	0.85	0.22	0.35	0.28
	Slowdown	0.90	-0.08	0.93	0.05
	Share	1.00	-0.09	1.04	0.05
United Kingdom	1996-2005	2.25	1.23	0.65	0.37
	2006-2017	0.52	0.31	0.23	-0.02
	Slowdown	1.73	0.92	0.42	0.39
	Share	1.00	0.53	0.24	0.22
United States	1998-2005	2.53	1.21	1.16	0.16
	2006-2017	0.95	0.33	0.46	0.17
	Slowdown	1.57	0.88	0.70	-0.01
	Share	1.00	0.56	0.44	-0.00

Table 17: Sources-of-growth decomposition using the intangible-extended ("Analytical") dataset from EU-KLEMS 2019 (Stehrer et al. 2019). The "Analytical" dataset includes extra intangible capital in the growth of the capital stock, and updates output measures to account for the additional investment into intangible capital.

The 2019 release of KLEMS includes two databases for growth accounting. The first, termed "Statistical", is used in Table 2. The second, termed "Analytical", recomputes national accounting identities using the extended asset boundary (see Table 6). It is essential to use a database where all the accounting is redone consistently, because converting expenses into investment implies a change to several quantities, present on both sides of the growth accounting equation: investment, capital stocks, GDP, and the labor share (see e.g. Appendix B, Corrado et al. (2009), Crouzet & Eberly (2021) and Brynjolfsson et al. (2021)).

D Computation of selected contributions to the productivity slowdown

D.1 Contribution of trade to the productivity slowdown

Constantinescu et al. (2019) estimate the elasticity of industry-level labor productivity to backward linkages. Here, we compute the slowdown in the growth of backward linkages, and use Constantinescu et al.'s (2019) elasticities to estimate the contribution to the productivity slowdown. Because these elasticities are industry-level, an important question is whether we consider only manufacturing industries, as in Constantinescu et al.'s (2019) baseline, or if we also consider tradable services, as in Constantinescu et al.'s (2019) extension. The more industries we consider, the larger the aggregate impact.

Variable construction. The key variable in measuring Global Value Chain (GVC) integration is backward linkages (Hummels et al. 2001, Constantinescu et al. 2019), which starts by the construction of the matrix

$$Z = V(I - A)^{-1}E, \tag{23}$$

where V is an $MN \times MN$ matrix, with diagonal elements equal to the ratio of value added to gross output of N countries and M industries, A is the $MN \times MN$ matrix of intermediate consumption over gross output (such that column sums are the share of total intermediate consumption out of gross output for the respective country-industry), and E is a $MN \times MN$ matrix with diagonal elements equal to gross exports (see the online Appendix of Constantinescu et al. (2019) for details).

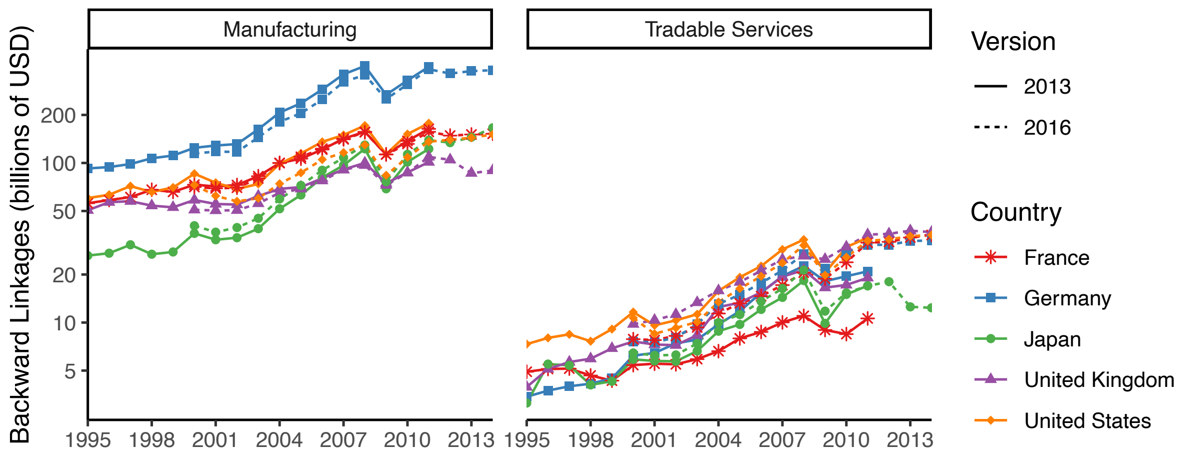

Figure 7: Foreign value added embodied in gross exports. The trends in the WIOD 2013 and WIOD 2016 databases are very similar for manufacturing industries, but substantially less so for service industries.

We construct the matrix Z using data from the World Input-Output Database (WIOD), for both the 2013 and 2016 vintages (Timmer et al. 2015, 2022). We then construct backward linkages $B_{i,j,t}$ for country i, industry j in year t, by fixing a column of Z and summing across rows all the elements for which the origin country (in the rows) is different from the destination country (in the column). Thus, $B_{i,j,t}$ is the foreign value added by country, industry and year, embodied in its gross exports (see the online Appendix of Constantinescu et al. (2019)). Figure 7 plots the total $B_{i,t}$ summed across all industries in a given year, for each of our five countries, and for each vintage of the WIOD. The industry selection is described later in this Appendix. In addition to their different different time coverage, the two vintages appear to have a small difference in the *level* of backward linkages, but very similar overall trends.

Contribution to the productivity slowdown. To derive an estimated contribution of a slowdown in backward linkages to the labor productivity slowdown, let us start from the analysis of Constantinescu et al. (2019), who estimate the impact of backward linkages on labor productivity levels using

$$\log y_{i,j,t} = \alpha X^c_{i,t} + \beta^{\mathrm{GVC}} \log B_{i,j,t} + \lambda_i + \lambda_j + \lambda_t + \varepsilon_{i,j,t}, \tag{24}$$

where labor productivity y is in value added per employee, controls X^c include log capital per worker and log gross final imports, and fixed effects λs are included for country i, industry j, and year t. The main variable of interest, $\log B$, is the log of foreign value added embodied in gross exports, which we derived previously. Constantinescu et al. (2019) estimate their specification using data from the 2013 vintage of the WIOD, for 40 countries, 13 manufacturing industries, and the years 1995-2009.

In order to use the estimated elasticity β^{GVC}, we aggregate the relevant industries for each year. For simplicity, we aggregate country-level labor productivity growth as

$$\Delta \log y_{i,t} = \sum_{j \in \mathcal{M}_1} v_{i,j,t} \Delta \log y_{i,j,t} + \sum_{j' \in \mathcal{M}_2} v_{i,j',t} \Delta \log y_{i,j',t},$$

which is the aggregated sum of m_1 "tradable" industries in the set $\mathcal{M}_1$, and m_2 "other" industries in the set $\mathcal{M}_2$ ($m_1 + m_2 = M$), and we use the Törnqvist indices

$$v_{i,j,t} = \frac{1}{2}\Big(Q_{i,j,t}/Q_{i,t} + Q_{i,j,t-1}/Q_{i,t-1}\Big),$$

where $Q_{i,j,t}$ is the nominal value added of industry j, country i at time t, and $Q_{i,t}$ is the aggregate nominal value added of country i at time t. Note that $\sum_j v_{i,j,t} + \sum_{j'} v_{i,j',t} = 1$.

From the first-difference version of Eq. 24, the contribution of the growth of backward linkages to productivity growth in industry j, which we denote $\Delta \log y^E_{i,j,t}$ is

$$\Delta \log y^E_{i,j,t} = \beta^{\mathrm{GVC}} \Delta \log B_{i,j,t}. \tag{25}$$

Note that industries with negative, or zero, gross exports are omitted after taking the log-transform. Defining an aggregate over the relevant industries only, and using Eq. 25, we have

$$\Delta \log y^E_{i,t} \equiv \sum_{j \in \mathcal{M}_1} v_{i,j,t} \Delta \log y^E_{i,j,t} = \beta^{\mathrm{GVC}} \sum_{j \in \mathcal{M}_1} v_{i,j,t} \Delta \log B_{i,j,t}. \tag{26}$$

The sum on the RHS is what we report as "Backward linkages" in Table 9. More precisely, we average this sum over the relevant years.

We compute this sum using the 2013 vintage only when it is the only one available, using the 2016 vintage only when it is the only available, and using an average of the two when both are available. From Fig. 7, we do not expect large differences between vintages. Across all countries, the correlation coefficient of backward linkages in manufacturing alone is 0.86 between the 2013 and 2016 releases, and 0.62 in manufacturing plus tradable service industries. When taking our five countries in isolation, the coefficients are 0.98 for manufacturing but only 0.34 for manufacturing plus tradable services, which is why we prefer to average over the two databases when possible.

Constantinescu et al. (2019) deflate their variable, but here we omit this step as this is unlikely to strongly affect the calculations for the contribution to the *slowdown* of productivity. To obtain the "Productivity effect" in Table 9, which is the LHS of Eq. 26, we have to make two choices: the value of β^{GVC}, and the set of industries over which we aggregate ($\mathcal{M}_1$).

Choice of industries. Constantinescu et al. (2019) consider only manufacturing industries in their baseline, but add tradable services in a robustness analysis. If backward linkages have slowed down in all industries, the more industries we consider, the stronger our estimated effect. We take a lower-bound scenario with manufacturing industries only, and an upper bound scenario with manufacturing and tradable services industries. We calculate backward linkages using the industries included in the regression analysis of Constantinescu et al. (2019), which are denoted in bold and italics in their Table A2. This is straightforward when computing aggregates from the 2013 vintage. For the 2016 vintage, we pick industries corresponding to those listed by Constantinescu et al. (2019) using the concordance table provided in the WIOD manual accompanying the database (Gouma et al. 2018, Section 5).

Choice of $\beta^{\mathbf{GVC}}$. In their preferred specification, Constantinescu et al. (2019) use an instrumental variable for log backward linkages, which averages value added from Germany, Japan and the United States, embodied in exports of three countries that are closest in income per capita to country i in question. In this specification, seen in columns 4 and 7 of their Table 2, they provide an estimate of $\beta^{\mathrm{GVC}} = 0.159(0.042)$ when considering manufacturing industries only, and $\beta^{\mathrm{GVC}} = 0.245(0.135)$ when considering manufacturing and tradable services. These are the largest coefficients they report. In other specifications, they find elasticities as low as $\beta^{\mathrm{GVC}} = 0.0338(0.0130)$ (column 6). Because there are large uncertainties, and our goal is to try to find upper and lower bounds rather than precise estimates, we apply the lowest coefficient in the manufacturing-only case, and the highest coefficient in the manufacturing plus tradable services case. This provides a reasonable best and worst case contribution of trade to the slowdown, with the exception of Japan where there has been a perceptible *acceleration* of the growth of linkages when considering Manufacturing only.

Finally, in Eq. 3 and in the summary table in the Conclusion, we consider that the contribution of trade to the productivity slowdown is through TFP. This is of course debatable, but we note that Eq. 24 used by Constantinescu et al. (2019) controls for capital per employee, so that we can also think of it as an estimate of the contribution of trade to a production function-based estimate of TFP.

D.2 Contribution of allocative efficiency to the TFP slowdown

Baqaee & Farhi (2020) introduce a decomposition of markup-corrected TFP into two terms: a term (itself composed of two terms) that relates to changes in allocative efficiency, and a residual. Baqaee & Farhi's (2020) model is a general equilibrium model with an input-output structure and exogenous distortions, modelled as markups.

To implement their model empirically, Baqaee & Farhi (2020) estimate firm-level markups (using three different methods), and assume that firms in the same sector have the same production function, up to the Hicks-neutral TFP shifters, allowing them to use sector-level input-output tables. If markups are aggregated adequately (i.e. as harmonic averages), the firm-level model can then be implemented at the sectoral level directly.

Here we take sector-level markups from the replication files of Baqaee & Farhi (2019*b*), and re-implement their sector-level derivation of the growth accounting results. This allows us to obtain year-specific decompositions which we need to estimate the contribution of allocative efficiency to the TFP *slowdown*, rather than to cumulative TFP growth as in the original paper.

Baqaee & Farhi (2020) implement their decomposition empirically as follows. We assume that there are two factors, labor and capital, and we assume that payments to labor are observable directly but payments to capital are not observable directly, because Gross Operating Surplus (GOS) includes pure profits and "normal" payments to capital.

Under constant returns to scale, marginal and average costs are the same, so price is the markup times the average cost per unit $P = \mu \frac{TC}{Y}$, denoting Total Costs by TC. Since total profits are defined as total sales minus total costs, $\pi = PY - TC$, we have

$$\mu = \frac{1}{1 - \alpha_\pi}, \tag{27}$$

where $\alpha_\pi \equiv \frac{\pi}{PY}$ is the share of profits in *sales*.

Now, if we define

$$\text{GOS} = PY - (\text{Intermediate costs} + wL) = \text{VA} - wL = \pi + rK,$$

where VA is Value Added and rK is the user cost of capital, we have $\frac{\text{GOS}}{PY} = \frac{\pi}{PY} + \frac{rK}{PY}$. If we define the share of capital costs in sales as $\alpha_K = \frac{rK}{PY}$, then using Eq. 27, we have

$$\alpha_K = \frac{\text{GOS}}{PY} - \left(1 - \frac{1}{\mu}\right). \tag{28}$$

We estimate α_K using Eq. 28, where GOS is line V003 in the BEA Tables ("Gross Operating Surplus") and PY is Gross Output (column "Total Commodity Output"), and μ is a vector of sales weighted industry-level (harmonic) average markups. Finally $\alpha_L = \frac{wL}{PY}$ is computed by reading wL directly from line V001 "Compensation of employees" (Note that the line V002 "Taxes on production and imports, less subsidies" is not considered).

If α_K, α_L are the shares of factors into sales, we can easily define the shares of factors and profits into total costs ,

$$\tilde{\alpha}_K = \frac{rK}{\text{TC}} = \frac{rK}{PY/\mu} = \mu \alpha_K, \tag{29}$$

$$\tilde{\alpha}_L = \mu \alpha_L. \tag{30}$$

We can construct a $(N+F) \times (N+F)$ matrix where on a line i, the first N entries show the intermediate expenses and the last F entries show the factor expenses of producer i. The row sums of this matrix are the total costs of producers, and the column sums are the total sales of the producers. Crucially, these vectors differ in general, because of pure profits/markups. We denote by $\tilde{\Omega}$ the row-normalized version of this matrix, where an entry $\tilde{\Omega}_{ij}$ is the share of j (which is either an intermediate input or a factor) into i's total cost. We use the notation

$$\tilde{\Omega} = \left[\begin{array}{c|c} \tilde{\Omega}^p & \tilde{\Omega}^f \\ \hline 0 & 0 \end{array}\right]$$

to distinguish parts of the matrix relating to intermediates and to factors. The $N \times 2$ matrix of shares of factors into costs simply concatenates the column vectors defined in Eqs. 29-30,

$$\tilde{\Omega}^f = [\tilde{\alpha}_K \tilde{\alpha}_L]. \tag{31}$$

Now, to get $\tilde{\Omega}^p$ from the BEA Input-Output tables, we take the "Use of Commodities by Industries, Before Redefinitions (Producers' Prices)" table, transposed, and keep only $N = 66$ industries as in Baqaee & Farhi (2020). This gives the $N \times N$ table X where X_{ij} are the expenses of producer i on a product sold by j.

If we row-normalize X to define $\bar{X}$, we have $\bar{X}_{ij} = \frac{X_{ij}}{\mathrm{IC}_i}$, where IC_i is the total intermediate cost of i. Thus, by definition of $\tilde{\Omega}_{ij}$, we have

$$\tilde{\Omega}_{ij} = \frac{X_{ij}}{\mathrm{TC}_i} = \bar{X}_{ij}\frac{\mathrm{IC}_i}{\mathrm{TC}_i}. \tag{32}$$

By definition $\mathrm{IC}_i = \mathrm{TC}_i - rK_i - wL_i$. Dividing this through by TC_i, using $rK_i/\mathrm{TC}_i = \tilde{\alpha}_{K_i}$ from Eq. 29 (and similarly for labor), and rearranging, we have

$$\frac{\mathrm{IC}_i}{\mathrm{TC}_i} = 1 - \tilde{\alpha}_{K_i} - \tilde{\alpha}_{L_i}, \tag{33}$$

so that substituting Eq. 33 into 32, we have $\tilde{\Omega}_{ij} = \bar{X}_{ij}(1-\tilde{\alpha}_{K_i}-\tilde{\alpha}_{L_i})$, which in matrix form reads

$$\tilde{\Omega}^p = \mathrm{diag}(1-\tilde{\alpha}_K - \tilde{\alpha}_L)\bar{X}. \tag{34}$$

The revenue-based Input-Output matrix, which gives the share of producer j in i's *sales*, is related to $\tilde{\Omega}^p$ by

$$\Omega^p = \mathrm{diag}(1/\mu)\tilde{\Omega}^p. \tag{35}$$

Similarly for factors (in practice labor and capital),

$$\Omega^f = \mathrm{diag}(1/\mu)\tilde{\Omega}^f. \tag{36}$$

We define the $N \times 1$ vector b as the share of an industry in final demand $b_i = \frac{p_i y_i}{\mathrm{GDP}}$, which we read from the column "Total Final Uses (GDP)" of the BEA tables.

Now that we have $\tilde{\Omega}^p$ (Eq. 34) and $\tilde{\Omega}^f$ (Eq. 31), we can define their Leontief inverses $\tilde{\Psi}^p = (I-\tilde{\Omega}^p)^{-1}$ and $\Psi^p = (I-\Omega^p)^{-1}$. From these we can obtain all the cost- and revenue-based Domar weights for intermediates and for factors, needed for the decomposition. The revenue based Domar weights for intermediates, $\lambda_i = \frac{p_i y_i}{\mathrm{GDP}}$, are actually not needed but it is interesting to note that one can show $\lambda = b'\Psi^p$. Similarly, the factor shares $\frac{rK}{\mathrm{GDP}}$ and $\frac{wL}{\mathrm{GDP}}$ are equal to

$$\Lambda = b'\Psi^p\Omega^f. \tag{37}$$

Note that these do not sum up to 1, since income is also allocated to pure profits.

The *cost-based Domar weights* are given by

$$\tilde{\lambda} = b'\tilde{\Psi}^p, \tag{38}$$

and the *cost-based factor shares* (which do sum up to 1) are

$$\tilde{\Lambda} = b'\tilde{\Psi}^p\tilde{\Omega}^f. \tag{39}$$

Let us assume that we observe output growth $\Delta \log Y_t$, and the vector of inputs growth $\Delta \log \mathcal{L}_t = [\Delta \log \bar{L}_t, \Delta \log K_t]$, where $\bar{L}_t$ is composition-adjusted labor inputs, and K is capital services. Then, using the quantities defined in Eqs. 37, 38, 39, together with the markups μ, we can perform the decomposition (Proposition 1 in Baqaee & Farhi (2020), Eq. 8 in the main text, reproduced here for convenience)

$$\underbrace{\Delta \log Y_t - \tilde{\Lambda}'_{t-1}\Delta \log \mathcal{L}_t}_{\Delta\text{ Markup-corrected Solow residual}} \approx \underbrace{\tilde{\lambda}'_{t-1}\Delta \log \mathcal{A}_t}_{\Delta\text{ Technology}} \underbrace{-\tilde{\lambda}'_{t-1}\Delta \log \mu_t - \tilde{\Lambda}'_{t-1}\Delta \log \Lambda_t}_{\Delta\text{ Allocative Efficiency}},$$

by computing the LHS and the last two terms of the RHS corresponding to changes in allocative efficiency. The first term on the RHS, corresponding to the change in "Technology", is estimated as a residual.

To perform the decomposition, Baqaee & Farhi (2020) use inputs and output growth data from Fernald (2014), and we reuse these. We reproduced exactly Fig. IV, A.1.A and A.2.A in Baqaee & Farhi (2020), and also checked the results of the decomposition when assuming $\Delta \log \mu = 0$. In this case, if we fix μ at its initial value in 1997, allocative efficiency makes a very small negative contribution. If we fix $\mu = 1$, allocative efficiency makes no contribution, as expected.

References

Baily, M. N., Bosworth, B. P. & Doshi, S. (2020), 'Lessons from productivity comparisons of Germany, Japan, and the United States', *International Productivity Monitor* **38**, 81–103.

Baily, M. N. & Montalbano, N. (2016), 'Why is US productivity growth so slow? Possible explanations and policy responses', *Brookings Institution, Hutchins Center Working Paper* **22**.

Baqaee, D. R. & Farhi, E. (2019), 'Replication Data for: 'Productivity and Misallocation in General Equilibrium''. **URL:** *https://doi.org/10.7910/DVN/CT5MZN*

Baqaee, D. R. & Farhi, E. (2020), 'Productivity and misallocation in general equilibrium', *Quarterly Journal of Economics* **135**(1), 105–163.

Bergeaud, A., Cette, G. & Lecat, R. (2016), 'Productivity trends in advanced countries between 1890 and 2012', *Review of Income and Wealth* **62**(3), 420–444.

Bergeaud, A., Cette, G. & Lecat, R. (2021), Long term productivity database, Dataset v. 2.4, ''. http://www.longtermproductivity.com/download/BCLDatabase_online_v2.4.xlsx (accessed May 2021).

Brynjolfsson, E., Rock, D. & Syverson, C. (2021), 'The productivity J-curve: How intangibles complement general purpose technologies', *American Economic Journal: Macroeconomics* **13**(1), 333–72.

Cette, G., Fernald, J. & Mojon, B. (2016), 'The pre-great recession slowdown in productivity', *European Economic Review* **88**, 3 – 20.

Constantinescu, C., Mattoo, A. & Ruta, M. (2019), 'Does vertical specialisation increase productivity?', *The World Economy* **42**(8), 2385–2402.

Corrado, C., Haskel, J., Jona-Lasinio, C. & Iommi, M. (2016), 'Intangible investment in the EU and US before and since the great recession and its contribution to productivity growth', *EIB Working Papers* **2016-08**.

Corrado, C., Hulten, C. & Sichel, D. (2009), 'Intangible capital and US economic growth', *Review of Income and Wealth* **55**(3), 661–685.

Crouzet, N. & Eberly, J. (2021), 'Intangibles, markups, and the measurement of productivity growth', *Journal of Monetary Economics* **124**, S92–S109.

Fernald, J. (2014), 'A quarterly, utilization-adjusted series on total factor productivity', *Federal Reserve Bank of San Francisco WP* **2012-19**.

Fernald, J. & Inklaar, R. (2020), 'Does disappointing european productivity growth reflect a slowing trend? weighing the evidence and assessing the future', *International Productivity Monitor* **2020**(38), 104–135.

Fouquet, R. & Broadberry, S. (2015), 'Seven centuries of european economic growth and decline', *Journal of Economic Perspectives* **29**(4), 227–44.

Goodridge, P., Haskel, J. & Wallis, G. (2018), 'Accounting for the UK productivity puzzle: A decomposition and predictions', *Economica* **85**(339), 581–605.

Gordon, R. J. & Sayed, H. (2019), The industry anatomy of the transatlantic productivity growth slowdown, Working Paper 25703, National Bureau of Economic Research.

Gouma, R., Chen, W., Woltjer, P. & Timmer, M. (2018), 'WIOD socio-economic accounts 2016: Sources and methods', *World Input-Output Database Project* . Available at http://www.wiod.org/protected3/data16/SEA/SEA16_Sources.pdf.

Hummels, D., Ishii, J. & Yi, K.-M. (2001), 'The nature and growth of vertical specialization in world trade', *Journal of International Economics* **54**(1), 75–96.

Inklaar, R., Jäger, K., O'Mahony, M. & van Ark, B. (2019), European productivity in the digital age: evidence from EU KLEMS, *in* B. M. Fraumeni, ed., 'Measuring Economic Growth and Productivity: Foundations, KLEMS Production Models, and Extensions', 1st edn, Elsevier, p. 75–94.

Jorgenson, D. W., Nomura, K. & Samuels, J. D. (2018), Progress on measuring the industry origins of the Japan-U.S. productivity gap, Working paper, Harvard University.

Koh, D., Santaeulàlia-Llopis, R. & Zheng, Y. (2020), 'Labor share decline and intellectual property products capital', *Econometrica* **88**(6), 2609–2628.

Murray, A. (2018), 'What explains the post-2004 U.S. productivity slowdown?', *International Productivity Monitor* **34**, 81–109.

OECD (2021*a*), Productivity and ULC by main economic activity (ISIC rev.4), Dataset Accessed April 2021, OECD. https://stats.oecd.org, Productivity and ULC - Annual, Total Economy.

OECD (2021*b*), Structural analysis database, Dataset OECD STAN database, Accessed February 2021, OECD. https://stats.oecd.org, Industries and Services > STAN.

Oulton, N. (2019), 'The UK and western productivity puzzle: Does Arthur Lewis hold the key?', *International Productivity Monitor* **36**, 110––141.

Riley, R., Rincon-Aznar, A. & Samek, L. (2018), 'Below the Aggregate: A Sectoral Account of the UK Productivity Puzzle', *ESCoE Discussion Paper* **2018-06**.

Romer, P. M. (1986), 'Increasing returns and long-run growth', *Journal of Political Economy* **94**(5), 1002–1037.

Stehrer, R., Bykova, A., Jäger, K., Reiter, O. & Schwarzhappel, M. (2019), Industry Level Growth and Productivity Data with Special Focus on Intangible Assets, Report, The Vienna Institute for International Economic Studies.

Syverson, C. (2017), 'Challenges to mismeasurement explanations for the US productivity slowdown', *Journal of Economic Perspectives* **31**(2), 165–86.

Tang, J. & Wang, W. (2004), 'Sources of aggregate labour productivity growth in Canada and the United States', *The Canadian Journal of Economics / Revue canadienne d'Economique* **37**(2), 421–444.

Tenreyro, S. (2018), 'The fall in productivity growth: causes and implications'.
URL: *https://www.bankofengland.co.uk/speech/2018/silvana-tenreyro-2018-peston-lecture*

The Conference Board (2020), Total economy database$^{\mathrm{TM}}$, Dataset July 2020, The Conference Board. https://www.conference-board.org/data/economydatabase/total-economy-database-productivity.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R. & De Vries, G. J. (2015), 'An illustrated user guide to the world input–output database: the case of global automotive production', *Review of International Economics* **23**(3), 575–605.

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R. & de Vries, G. J. (2022), World input-output database (wiod) [dataset], Dataset v. 2013 and 2016, 'University of Groningen'. https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release and https://www.rug.nl/ggdc/valuechain/wiod/wiod-2016-release(accessed April 2022).

van Ark, B. (2016), Europe's productivity slowdown revisited: A comparative perspective to the United States, *in* P. Askenazy, L. Bellmann, A. Bryson & E. M. Galbis, eds, 'Productivity Puzzles Across Europe', Oxford Scholarship Online.